Asymptotic Factorization of the n-particle SU(N) Form Factors

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11-18.08.2024(14.08-21.08.,2022),Varna

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Motivation

- In QCD the calculation of the structure functions for all values of the Bjorken variable x is still an open problem.
- On other side, the existence of exact integrable models in 1+1 dimensional asymptotically free theories may be relevant, providing valuable insights into this discussion.
- In the remarkable papers Balog and Weisz(BW-2004) define analogs of the structure functions in two-dimensional integrable quantum field theories.

The Motivation.

- The Bjorken Scattering or inelastic lepton-hadron scattering at high energies has been a very important and crucial stage in the development of modern QCD(J. Bjorken , R. Feynman).

- The essential point in these studies is the behavior of the structure functions of the hadrons(R. Feynman).

They describe the parton (quark) structure of the hadrons and the nature of the interaction between the quarks inside of the hadrons.

The amplitude of the lepton-hadron interaction consists of two parts, where the lepton part is well known.

- The hadron part , whose invariant decomposition provides the form factors or structure functions , is not known.

Motivation

- I. the example of the O(N) sigma model they(BW) consider form factors of the current operator(related with structure function) which are accurately computed over the whole x range.
- Also authors employ the so called cluster behavior(factorization) of the for factors to calculate the same structure functions.
- Here we learn how to calculate the different asymptotic behavior of the form factors in SU(N) Gross-Neveu model.

Motivation and Structure functions

- We should point out that the first investigation of the cluster behavior(factorization) of the exact form factors was performed by F . Smirnov in case of the sine-Gordon, SU(2) Thirring model and O(3) sigma model

- The structure functions are define by Balog-Weisz as follows
- $W_{\boldsymbol{\mu\nu}}{}^{ab;cdef}(p,q) =$
- $\frac{1}{4\boldsymbol{\pi}}\int d^2x e^{iqx} < a,p|[J_{\boldsymbol{\mu}}{}^{cd}(\mathrm{x}),J_{\boldsymbol{\nu}}{}^{ef}(o)|b,p >$

The Bjorken variable

$$X=-\frac{q^2}{2(pq)}$$

The aim of Form Factor Program .

- If we know the exact two particle S–matrix, it means we in principle know the in and out states of the theory . If we know the in and out states then one can calculate the exact form factors of the local field $\boldsymbol{\varphi}(\mathrm{x})$ in 1+1 dimensional exact integrable QFT (A. Polyakov)
- $< o \mid \boldsymbol{\varphi}(\mathrm{x}) \mid \boldsymbol{\theta}_1 , \boldsymbol{\theta}_2 ,\ldots\ldots \boldsymbol{\theta}_N >$(in or out)
- First papers was Vergiles, Gryanik-in sinh-Gordon model and A. Zamolodchikov in sine-Gordon model.
- M. Karowski and P.Weisz formulate the form factor program or bootstrap program. Final equations for form factors formulated by F. Smirnov.

The Form Factor Program

- The bootstrap program for integrable quantum field theories in 1+1 dimension
- classifies integrable quantum field theoretic models and in addition it provides their explicit exact solutions in term of all Wightman functions. The results are obtained in three steps
- 1.The S-matrix is calculated by means of general properties such as unitarity and crossing, Yang-Baxter equations and additional assumption of maximal analyticity.
- This means that two particle S-matrix is an analytic function in the physical plane

The Form Factor Program

- And possesses only those poles there which are physical origin. The only input which depends on the model is the assumption of the particle spectrum . Usually it belongs representations of symmetry.
- 2.Generalized form factors which are matrix elements of local operators
- out$< p_m{}', \ldots p_1{}' | \boldsymbol{\varphi}(\mathrm{x}) | p_1, \ldots p_n > in$
- are calculated by means of the S matrix . More precisely, the equations (i)-(V), given below
- are solved . These equations follow from LSZ – assumptions and again the additional “assumption analyticity”.

The Form Factor Program

- 3.The Wightman functions are obtained by inserting a complete set of intermediate states . In particular the two point function
- for a hermitian operator $\boldsymbol{\varphi}$(x) reads

- $< 0 \mid \boldsymbol{\varphi}(x)\boldsymbol{\varphi}(0) \mid >= \sum_{n=0}^{\infty} \frac{1}{n!} \int \frac{dp_1}{2\boldsymbol{\omega}_1} \ldots \int \frac{dp_n}{2\boldsymbol{\omega}_n} *$
- $|< 0 \, |\boldsymbol{\varphi}(0)| p_1, \ldots p_n > in|$^2 exp(-ix$\sum p_i$)

- Up to now proof that these sums converge does not exists.

The Form Factor Equations

- Form factor equations formulated in final form by F.Smirnov. We will call them as Karowski-Weisz-Smirnov Equations.
- First introduce the notation
- $< o \mid \boldsymbol{\varphi}(0) \mid \boldsymbol{\theta}_1 , \boldsymbol{\theta}_2, \ldots\ldots \boldsymbol{\theta}_N > = \boldsymbol{\varphi}(\boldsymbol{\theta}_1 , \boldsymbol{\theta}_2, \ldots\ldots \boldsymbol{\theta}_N)$
- (i)Watson Equations
- $\boldsymbol{\varphi}(\ldots\boldsymbol{\theta}_i, \boldsymbol{\theta}_j, \ldots.) = \boldsymbol{\varphi}(\ldots\boldsymbol{\theta}_j, \boldsymbol{\theta}_i, \ldots.) \mathrm{S}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)$
- (ii)Crossing relations
- $\boldsymbol{\varphi}(\boldsymbol{\theta}_1 + i\boldsymbol{\pi} , \boldsymbol{\theta}_2, \ldots\ldots \boldsymbol{\theta}_N) = \boldsymbol{\varphi}(, \boldsymbol{\theta}_2, \ldots\ldots \boldsymbol{\theta}_N, \boldsymbol{\theta}_1 - i\boldsymbol{\pi})$
- (iii) Recursion relations

  $\mathrm{Res}(\boldsymbol{\theta}_{12} = i\boldsymbol{\pi})\boldsymbol{\varphi}(\boldsymbol{\theta}_1 , \boldsymbol{\theta}_2, \ldots\ldots \boldsymbol{\theta}_N) = 2\mathrm{i}C_{12}\boldsymbol{\varphi}(, \boldsymbol{\theta}_3, \ldots\ldots \boldsymbol{\theta}_N)(1 - S_{2n}, \ldots S_{23})$
- (iV) Bound state form factor equations

The Form Factor Equations

- If there are also bound states in the model the function $\boldsymbol{\varphi}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots \boldsymbol{\theta}_N)$ has additional poles. For instance the particle 1 and 2 form bound state (12),there is a pole at
- $\boldsymbol{\theta}_{12} = \boldsymbol{\eta}$ $(0 < \boldsymbol{\eta} < \boldsymbol{\pi})$.
- $\mathrm{Res}(\boldsymbol{\theta}_{12} = \boldsymbol{\eta})\boldsymbol{\varphi}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots \boldsymbol{\theta}_N) =$
- $\boldsymbol{Y}^{(12)}\boldsymbol{\varphi}(\boldsymbol{\theta}_{(12)}, \boldsymbol{\theta}_3, \ldots \boldsymbol{\theta}_N)$
- Where $\boldsymbol{Y}^{(12)}$ is the bound state intertwiner.
- (V) Naturally, since we dealing with relativistic quantum field theories we finally have

The S-matrix of the SU(N) Gross-Neveu model.Particle content.

$\boldsymbol{\varphi}(\boldsymbol{\theta}_1 + \boldsymbol{\mu}, \boldsymbol{\theta}_2 + \boldsymbol{\mu}, \ldots\ldots \boldsymbol{\theta}_N + \boldsymbol{\mu}) =$
$\mathrm{Exp}\,(s\boldsymbol{\mu})\boldsymbol{\varphi}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots\ldots \boldsymbol{\theta}_N)$

Where s is the spin of the local field.

- The two particle $S-matrix$ is $S_{\boldsymbol{\alpha\beta}}{}^{\boldsymbol{\delta\gamma}}(\boldsymbol{\theta}) = \boldsymbol{\delta_\alpha}{}^{\boldsymbol{\gamma}}\boldsymbol{\delta_\beta}{}^{\boldsymbol{\delta}}\mathrm{b}(\boldsymbol{\theta}) + \boldsymbol{\delta_\alpha}{}^{\boldsymbol{\delta}}\boldsymbol{\delta_\beta}{}^{\boldsymbol{\gamma}}\mathrm{c}(\boldsymbol{\theta})$

Where $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta} = 1, \ldots N$ denote fundamental particles. We also introduce

$$\overline{S_{\boldsymbol{\alpha\beta}}}{}^{\boldsymbol{\delta\gamma}}(\boldsymbol{\theta}) = S_{\boldsymbol{\alpha\beta}}{}^{\boldsymbol{\delta\gamma}}(\boldsymbol{\theta})/\mathrm{a}(\boldsymbol{\theta}) = \boldsymbol{\delta_\alpha}{}^{\boldsymbol{\gamma}}\boldsymbol{\delta_\beta}{}^{\boldsymbol{\delta}}\mathrm{b}(\boldsymbol{\theta})/\mathrm{a}(\boldsymbol{\theta}) + \boldsymbol{\delta_\alpha}{}^{\boldsymbol{\delta}}\boldsymbol{\delta_\beta}{}^{\boldsymbol{\gamma}}\mathrm{c}(\boldsymbol{\theta})/a(\boldsymbol{\theta})$$

$$\mathrm{a}(\boldsymbol{\theta}) = \mathrm{b}(\boldsymbol{\theta}) + \mathrm{c}(\boldsymbol{\theta})) = \frac{\boldsymbol{\Gamma}\left(\frac{\boldsymbol{\theta}}{2i\boldsymbol{\pi}}\right)\boldsymbol{\Gamma}\left(1 - \frac{1}{N} + \frac{\boldsymbol{\theta}}{2i\boldsymbol{\pi}}\right)}{\boldsymbol{\Gamma}\left(\frac{\boldsymbol{\theta}}{2i\boldsymbol{\pi}}\right)\boldsymbol{\Gamma}\left(1 - \frac{1}{N} - \frac{\boldsymbol{\theta}}{2i\boldsymbol{\pi}}\right)}$$

The S-matrix of the SU(N) Gross-Neveu model.

$\overline{b}\,(\boldsymbol{\theta}) = \frac{b(\boldsymbol{\theta})}{a(\boldsymbol{\theta})} = \frac{\boldsymbol{\theta}}{\boldsymbol{\theta}-i\boldsymbol{\eta}}$, $\mathrm{C}(\boldsymbol{\theta}) = \frac{c(\boldsymbol{\theta})}{a(\boldsymbol{\theta})} = \frac{-i\boldsymbol{\eta}}{\boldsymbol{\theta}-i\boldsymbol{\eta}}$, $\boldsymbol{\eta} = \frac{2\boldsymbol{\pi}}{N}$

This S-matrix satisfy to Yang-Baxter equation and in addition unitarity also crossing relation.

The particle spectrum of the chiral SU(N) Gross-Neveu model consists of N-1 multiplets of particles of mass

$$m_r = m_1 \frac{\sin(\frac{r\boldsymbol{\pi}}{N})}{\sin(\frac{\boldsymbol{\pi}}{N})}$$

The particle spectrum of SU(N) chiral Gross–Neveu model.

Which correspond to all fundamental SU(N) representations of the rank r=1,2,…N–1 with representation spaces $V^{(r)}$ of dimension D= $\binom{N}{r}$

Let $(\boldsymbol{\alpha})=(\boldsymbol{\alpha}_1,\ldots\boldsymbol{\alpha}_r)$, $(1\leq \boldsymbol{\alpha}_1 < \boldsymbol{\alpha}_2 < \cdots < \boldsymbol{\alpha}_r \leq N)$ be particle of rank r. We write

$(\boldsymbol{\alpha})\in V=\sum_{r=1}^{N-1}\bigoplus V^{(r)}$, $V^{(r)}\cong C^D$

Where $(\boldsymbol{\alpha})$ form basis of V. Particle of rank r is a bound state of r particle of rank 1. The antiparticle correspond to $(\boldsymbol{\alpha})$ is a $(\bar{\boldsymbol{\alpha}})=(\bar{\boldsymbol{\alpha}}_1,\ldots\bar{\boldsymbol{\alpha}}_{N-r})$,

$(1\leq \bar{\boldsymbol{\alpha}}_1 < \bar{\boldsymbol{\alpha}}_2 < \ldots\bar{\boldsymbol{\alpha}}_{N-r} \leq N)$

The Particle spectrum of SU(N) chiral Gross–Neveu model

(particle of rank N–r) such that the union of the set of indices satisfies

$\{\boldsymbol{\alpha}_1,\ldots\boldsymbol{\alpha}_r\}\cup\{\overline{\boldsymbol{\alpha}}_1,\ldots\overline{\boldsymbol{\alpha}}_{N-r}\}=\{1,\ldots N\}$

We believe it is also verified by direct calculation(1/N expansion) that form factors calculated using this SU(N) invariant S–matrix correspond to the quantum field theoretical model which described by Langrangian of the chiral SU(N) Gross–Neveu model

$$L=\sum_{i=1}^{N}\overline{\boldsymbol{\Psi}}_i\, i\boldsymbol{\gamma}\partial\boldsymbol{\Psi}_i+\frac{g^2}{2}\left(\left(\sum_{i=1}^{N}\overline{\boldsymbol{\Psi}}_i\boldsymbol{\Psi}_i\right)^2-\left(\sum_{i=1}^{N}\overline{\boldsymbol{\Psi}}_i\,\boldsymbol{\gamma}_5\boldsymbol{\Psi}_i\right)^2\right)$$

The solution of the form factor equations. Master formula.

- Minimal form factors.

To construct the form factors we need the minimal form factors $F(\theta)$ for two particles

$$F(\theta)=c\exp\left\{\int_0^\infty \frac{dt}{t(\sinh(t))^2}\, e^{\frac{t}{N}}\sinh\left(t\left(1-\frac{1}{N}\right)\right)\left(1-\cosh t\left(1-\frac{\theta}{i\pi}\right)\right)\right\}=\frac{G\left(\frac{\theta}{2i\pi}\right)G\left(1-\frac{\theta}{2i\pi}\right)}{G\left(1-\frac{1}{N}+\frac{\theta}{2i\pi}\right)G\left(2-\frac{1}{N}+\frac{\theta}{2i\pi}\right)},\qquad c=F(i\pi)=\frac{G\left(\frac{1}{2}\right)^2}{G\left(\frac{3}{2}-\frac{1}{N}\right)^2}$$

Where $G(z)$ is Barnes G-function. The $F(x)$ it is minimal solution of the Watson equations

$F(\theta)=F(-\theta)a(\theta)$, $F(i\pi-\theta)=F(i\pi+\theta)$, $a(\theta)=c(\theta)+b(\theta)$

The solution of the form factor equations.

- The function Φ satisfies

$\prod_{k=0}^{N-2} \Phi((-\theta-ik\boldsymbol{\eta}) = \prod_{k=0}^{N-1} F(\theta+ik\eta) = 1$

with the solution

$$\Phi(\theta) = \frac{1}{F(-\theta)\bar{F}(i\boldsymbol{\pi}+\theta)} = \Gamma(-\frac{\theta}{2i\boldsymbol{\pi}})\Gamma(1-\frac{1}{N}+\frac{\theta}{2i\boldsymbol{\pi}})$$

Where

$$\bar{F}(\theta) = \bar{c}\exp\{\int_0^\infty \frac{dt}{t(\sinh(t))^2} e^{\frac{t}{N}} \sinh(\frac{t}{N}(1-\cosh t(1-\frac{\theta}{2i\boldsymbol{\pi}}))\} = \frac{G(\frac{1}{2}-\frac{1}{N}+\frac{\theta}{2i\pi})G(\frac{3}{2}-\frac{1}{N}-\frac{\theta}{2i\pi})}{G(\frac{1}{2}+\frac{\theta}{2i\pi})G(\frac{3}{2}-\frac{\theta}{2i\pi})}, \bar{c} = \bar{F}(i\boldsymbol{\pi}) = G^2(1-\frac{1}{N})$$

The solution…

- Here we use other minimal form factor which is minimal form factor for a particle and an anti–particle satisfying to equation

$\bar{F}$(θ)=-$\bar{F}$(−θ)b(iπ-θ)

and also we define the τ –function

$$\tau(z)=\frac{1}{\Phi(z)\Phi(-z)}=\frac{1}{2\pi^2}\frac{z\sinh(\frac{z}{2})}{\Gamma(1-\frac{1}{N}+\frac{z}{2i\pi})\Gamma(1-\frac{1}{N}-\frac{z}{2i\pi})}$$

Using these functions we now are ready step by step construct the solution of the for factor equations.

The solution...

- n particle form factors:

The matrix element of a local operator O(x) for a state of n particles of kind α_i with rapidities θ_i

$$<0|O(x)|\theta_1,\ldots\theta_n >_{\underline{\alpha}}^{in}=\exp\{-ix(p_1,\ldots p_n\}F_{\underline{\alpha}}^{O}(\underline{\theta})$$

Defines the generalized for factor $F_{1,2,\ldots n}{}^{O}(\underline{\theta})$, which is a co-vector valued function with components $F_{\underline{\alpha}}{}^{O}(\underline{\theta})$. The form factors satisfy the form factor equations . Solutions of these equations can be written as follows:

The solution....

- As usual we split off the minimal part

$$F_{\underbrace{\alpha}}{}^{O}(\underbrace{\theta}) = N_n F(\underbrace{\theta}) K_{\alpha}{}^{O}(\underbrace{\theta}) \;,\; \mathrm{F}(\underbrace{\theta}) = \prod_{i<j}^{n} F(\theta_{ij})$$

where $\underbrace{\alpha} = (\alpha_1, \ldots \alpha_n)$, $\underbrace{\theta} = (\theta_1, \ldots \theta_n)$ and F(θ) is defined by Watson equation . The K-function is given by an "off-shell" Bethe Ansatz in terms of the multiple contour integral

$$K_{\underbrace{\alpha}}{}^{O}(\underbrace{\theta}) = \oint d\,\underbrace{z}\; \mathrm{h}(\underbrace{\theta}, \underbrace{z}) P^{O}(\underbrace{\theta}, \underbrace{z}) \psi_{\underbrace{\alpha}}(\underbrace{\theta}, \underbrace{z})$$

with $\underbrace{z} = (z_1, \ldots z_m)$ and $\oint d\,\underbrace{z} = \frac{1}{m!} \oint d\,z_1, \ldots \oint d\,z_m$

The solution…

- The integration contour is complicated and on can get in our paper . The scalar function h($\underbrace{\theta}\,\underbrace{z}$) depend only on the S–matrix and not on the specific operator O(x)

$$h(\underbrace{\theta}\,\underbrace{z})=\prod_{i=1}^{n}\prod_{j=1}^{m}\Phi(\theta_i - z_j)\prod_{i<j}^{m}\tau(z_i - z_j)$$

The dependence on the specific operator O(x) is encoded in the scalar p-function $P^{O}(\underbrace{\theta},\underbrace{z})$ which is in general a simple function of e^{θ_i} *and* e^{z_i}.

The solution…

- Bethe State.
- The state $\psi_{\underset{\sim}{\alpha}}$ in our solution is a linear combination of the basic Bethe Ansatz co-vectors

$$\psi_{\underset{\sim}{\alpha}}\left(\underset{\sim}{\theta}\,\underset{\sim}{z}\right) = L_{\underset{\sim}{\beta}}(\underset{\sim}{z})\varphi_{\underset{\sim}{\alpha}}^{\underset{\sim}{\beta}}\,(\underset{\sim}{\theta},\underset{\sim}{z}) \quad \text{with } 1 < \beta_i \leq N$$

as usual in the context of the algebraic Bethe ansatz the basic co-vectors are obtained from the monodromy matrix

The solution…

- $T_{1,2\ldots n,0}(\underbrace{\theta},z) = S_{10}(\theta_1 - z),\ldots S_{n0}(\theta_n - z) =$

$$\begin{pmatrix} A_{1,\ldots n}(\underbrace{\theta},z) & B_{1,\ldots n}(\underbrace{\theta},z) \\ C_{1,\ldots n}(\underbrace{\theta},z) & D_{1,\ldots n,\beta}{}^{\gamma}(\underbrace{\theta},z) \end{pmatrix}, \quad 2 \leq \beta,\gamma \leq N$$

The solution…

- Where the S-matrix is defined above and index 0 means the auxiliary space where are multiplied the S-matrices in definition of the monodromy matrix .The reference co-vector is defined as usual by $\omega B_{\beta}=0$, which implies

$\omega_{\underbrace{\alpha}}=\delta_{\alpha_1}{}^{1}, \ldots \delta_{\alpha_n}{}^{1}$

It is eigenstates of A and $D_{\beta}{}^{\gamma}$

$\omega A(\underbrace{\theta},z)=1, \quad \omega D_{\beta}{}^{\gamma}(\underbrace{\theta},z)=\delta_{\beta}{}^{\gamma}\prod_{i=1}^{n} b(\theta_i - z)\omega$

Where indices 1,2,....n are suppressed.

The solution…

- Then the basic Bethe ansatz co-vectors above are defined as

$$\varphi_{\underbrace{\alpha}}^{\underbrace{\beta}}(\underbrace{\theta}, \underbrace{z}) = (\omega C^{\beta_m}(\underbrace{\theta}, z_m) \dots C^{\beta_1}(\underbrace{\theta}, z_1))_{\underbrace{\alpha}}, \ 1 < \beta_i \leq N$$

The technique of the "nested Bethe Ansatz" means that for the coefficient $L_{\underbrace{\beta}}(\underbrace{z})$ in above formula one makes the analogous construction as for $K_{\underbrace{\alpha}}(\underbrace{\theta})$,where now the indices $\underbrace{\beta}$ take only the values 2< β_i ≤N. This nesting is repeated until the space

The solution…

- Of the coefficients becomes one dimensional.

 The final result is

$$K_{\underbrace{\alpha}}{}^{O}(\underbrace{\theta}) = \oint d\, \underbrace{\underbrace{z}}\; \mathrm{h}(\underbrace{\theta}, \underbrace{\underbrace{z}}) P^{O}(\underbrace{\theta}, \underbrace{\underbrace{z}}) \psi_{\underbrace{\alpha}}(\underbrace{\theta}, \underbrace{\underbrace{z}})$$

with the complete h-function

$$\mathrm{h}(\underbrace{\theta}, \underbrace{\underbrace{z}}) = \prod_{j=0}^{N-2} h(\underbrace{z_j}, \underbrace{z_{j+1}}), \qquad \underbrace{z_0} = \underbrace{\theta}$$

The solution…

- And with the complete Bethe ansatz state

$$\psi_{\underbrace{\alpha}}(\underbrace{\theta},\underbrace{\underbrace{z}})=(\psi^{N-2})_{\underbrace{\alpha}_{N-2}}^{\underbrace{\alpha}_{N-1}}(\underbrace{z}^{N-2},\underbrace{z}^{N-1})\dots$$

$$\dots(\psi^{1})_{\underbrace{\alpha}_{1}}^{\underbrace{\alpha}_{2}}(\underbrace{z}^{1},\underbrace{z}^{2})\ \psi_{\underbrace{\alpha}_{2}}^{\underbrace{\alpha}_{1}}(\underbrace{\theta},\underbrace{z}^{1})$$

where $\underbrace{\underbrace{z}}=(\underbrace{z}^{1},\dots\underbrace{z^{N-1}})$,$\underbrace{z}^{j}=(z_1^j,\dots z_{n_j}^j)$ and $\underbrace{\alpha_{N-1}}=(N,\dots N)$. It is well known that "off-shell" Bethe Ansatz states are highest weight states if they satisfy certain difference equations . If there are n particles the SU(N) weights are

$$w=(n-n_1,n_1-n_2,\dots n_{N-2}-n_{N-1},n_{N-1})=$$

Rapidity space clustering

- $=w^{O}+L(1,1,\ldots 1)$

where n_1=m,$n_2, n_3, \ldots n_{N-1}$ are the numbers of the C operators in the various levels of the nesting, w^{O} is the weigh vector of the operators O and L=1,2,… Note that w=(1,1,1… 1) correspond to vacuum sector.

- We shift k of the rapidities in the form factor $F^{O}_{\underbrace{\alpha}}(\underbrace{\theta})$

to ∞ and define

$\underbrace{\theta}_{w}=(\theta_1+W,\ldots\theta_k+W,\theta_{k+1},\ldots\theta_{k+l})=(\underbrace{\hat{\theta}}+W,\underbrace{\check{\theta}})$

Rapidity space clustering

- We investigate the behavior of the $F_{\underbrace{\alpha}}^{O}(\underbrace{\theta}_{w})$ for W→ ∞ .The result is of the form

$$F_{\underbrace{\alpha}}^{O}(\underbrace{\theta}_{w}) \to c_{\underbrace{\hat{O}}\underbrace{\check{O}}}{}^{O}(k,l,W) F_{\underbrace{\hat{\alpha}}}{}^{\hat{O}}(\underbrace{\hat{\theta}})\ F_{\underbrace{\check{\alpha}}}{}^{\check{O}}(\underbrace{\check{\theta}})$$

We calculate the functions $c_{\underbrace{\hat{O}}\underbrace{\check{O}}}{}^{O}(k,l,W)$ for several operators . Here we consider several fields which we will define in details with all quantum numbers.

Rapidity space clustering

- Examples of fields.

The SU(N) Noether current.

$J_a^{\mu} = \bar{\psi}_{\beta} \gamma^{\mu} \, (T_a)_{\alpha}^{\beta} \, \psi^{\alpha}$

Transforms as the adjoint representation with highest weights $w^J = (2,1,\ldots 1,0). The\ N^2-1$ generators of SU(N) satisfy

$[T_a, T_b] = i f_{abc} T_c$,tr$(T_a) = 0$,tr$(T_a T_b) = \frac{1}{2} \delta_{ab}$

The conservation law $\partial_{\mu} J_a^{\mu} = 0$, implies that J_a^{μ}(x) may be written in terms of the pseudo potential

J_a(x) as $J_a^{\mu}(x) = \varepsilon^{\mu\nu} \partial_{\nu} J_a(x)$

Rapidity space clustering

- With quantum numbers

Charge $Q^J = 0$

Weight vector $w^J = (2,1,\dots 1,0)$

Statistics factor $\sigma^J = 1$

Spin $S^J = 0$

Due the Swieca et all, the bound state of N−1 particles is to be identified with the anti−particle. This means that the anti −particle $\overline{\alpha}$ of a funsamental particle α of rank 1 is a bound state of rank N-1

Rapidity space clustering

- $\overline{\alpha}=(\rho)=(\rho_1,\ldots\rho_{N-1})$, with $\rho_1 <\ldots< \rho_{N-1}$, $\rho_i \neq \alpha$

The charge conjugation matrix is given by

$C_{\beta\overline{\alpha}}=C_{\beta\rho_1\rho_2,\ldots\rho_{N-1}}=C^{\overline{\alpha}\beta}=\varepsilon_{\beta\rho_{1,}\rho_2,\ldots\rho_{N-1}}$

with $C_{\beta\overline{\alpha}}C^{\overline{\alpha\gamma}}=\delta_{\beta}{}^{\gamma}$.In terms of fields this means

$\overline{\psi}_{\beta}=C_{\beta(\rho)}\psi^{\rho_1}\ldots\psi^{\rho_{N-1}}$.

For the current we also use

$J_{\mu}^{\alpha(\rho)}=\overline{\psi}^{(\rho)}\gamma_{\mu}\psi^{\alpha}-\frac{1}{N}C^{\alpha(\rho)}C_{(\sigma)\beta}\overline{\psi}^{(\sigma)}\gamma_{\mu}\psi^{\beta}$

$J_a = C_{\beta(\rho)}(T_a)_{\alpha}^{\beta}\,J^{\alpha(\rho)}$

Because the Bethe ansatz yields highest weight states we obtain the matrix element of highest weight component

Rapidity space clustering

- J(x)=$J^{1\bar{N}}(x) = J^{1(12\ldots N-1)}(x)$
- For the p-function of the current for the operator J we have
- $P^J(\underbrace{\theta}_{}\,\underbrace{\underbrace{z}_{}}_{})=\exp(\frac{i\pi n_1}{N})(\prod_{i=1}^{n}\exp(-\frac{\theta_i}{2}))(\prod_{i=1}^{n_1}\exp(\frac{{z_i}^{(1)}}{2}))*$
$*(\prod_{i=1}^{n_{N-1}}\exp(\frac{{z_i}^{(N-1)}}{2}))\frac{1}{\sum_{i=1}^{n}\exp(-\theta_i)}$
- The general weight formula of the Bethe states implies that the numbers of integrations in master formula satisfy
- n_j=n(1$-\frac{j}{N}$)−1, j=1,…N−1

Rapidity space clustering

- Energy momentum $T^{\mu\nu}$.

We write the energy momentum tensor in terms of an energy momentum potential

$T^{\mu\nu}=R^{\mu\nu}(i\partial_x)\mathrm{T}(\mathrm{x})$, $\quad R^{\mu\nu}(p)=-P^{\mu}P^{\nu}+g^{\mu\nu}P^2$

With

charge $Q^T=0$

weight vector $w^T=(0,\ldots0)$

statistics factor $\sigma^T=1$

spin $s^T=0$

We propose the p-function of the potential T(x)

$$P^T=P^{T_+}-P^{T_-}\ ,\ P^{T_\pm}=\frac{\sum e^{\pm z_j^{(1)}}}{\sum e^{\pm\theta_j}}$$

Rapidity space clustering

- The general weight formula of Bethe states implies that numbers of integrations are

$n_j = n(1-\frac{j}{N})$,j=1,…N−1

- The iso-scalar φ(x).
- With the quantum numbers
- charge Q^{φ}=o
- weight vector w^{φ}=(oo…o)
- statistics factor $\sigma^{\varphi}=e^{-i\eta}$
- spin s^{φ}=o

Rapidity space clustering

- And the p-function
- $P^{\varphi}(\underbrace{\theta}\ \underbrace{\underbrace{z}})=e^{\frac{i\pi n_1}{N}}(\prod_{i=1}^{n}\exp(-(1-\frac{1}{N})\theta_i))(\prod_{i=1}^{n_1}e^{z_i^{(1)}})$
- And with numbers of integrations
- $n_j=n(1-\frac{j}{N})$, j=1,…N-1

- The fundamental field $\psi^{\alpha}(x)$ of the chiral SU(N) Gross-Neveu model with the quantum numbers
- charge $Q^{\psi}=1$
- weight vector $\omega^{\psi}=(1,0,\ldots 0)$
- Statistics factor $\sigma^{\psi}=e^{i\pi\frac{(1-\frac{1}{N})}{2}}$
- spin $s^{\psi}=-\frac{1}{2}(1-\frac{1}{N})$

Rapidity space clustering

- The p–function of the highest component $\psi=\psi^1$ for n=1 mod N is
- $p^{\psi}=e^{\frac{in_1\eta}{2}}(\prod_{i=1}^{n} e^{-\frac{1}{2}(1-\frac{1}{N})\theta_i})(\prod_{i=1}^{n_1} e^{\frac{z_i^{(1)}}{2}})$
- The weight formula for this field is $\omega^{\psi} = (1,0 \dots 0)$
- wich implies that the numbers of integrations are
- $n_j = (n-1)(1-\frac{j}{N})$, j=1,…,N–1
- $1 - particle\ matrix\ element\ - F_{\alpha}{}^{\psi}(\theta)=\delta_{\alpha}^{1}e^{-\frac{1}{2}(1-\frac{1}{N})\theta}$

- Spinor $\chi^{\bar{\alpha}}(x)$ field.

  With the quantum numbers

Rapidity space clustering.

- charge $Q^{\bar{\chi}} = \mathrm{N-1}$
- weight vector $\omega^{\bar{\chi}} = (1,1,\ldots 1,0)$
- Statistics factor $\sigma_1^{\bar{\chi}} = e^{i\pi(N-\frac{1}{N})}$
- Spin $s^{\bar{\chi}} = \frac{1}{2}(1-\frac{1}{N})$
- The p-function of the highest weight component $\chi = \chi^{\bar{N}}$ for n=N-1 mod N is
- $P^{\bar{\chi}} = \frac{e^{i\eta(\frac{1}{2}(n_{N-1}+n_1))}}{\sum e^{-\theta_i}} (\prod_{j=1}^{n_1} e^{z_j^{(1)}})(\prod_{j=1}^{n_{N-1}} e^{z_j^{(N-1)}})$
- One particle matrix element $F_{\bar{\alpha}}^{\chi^{\bar{\beta}}}(\theta) = \delta_{\bar{\alpha}}^{\bar{\beta}}\, e^{\frac{1}{2}(1-\frac{1}{N})\omega}$

Results.

1.The result is of the form

- $F_{\underbrace{\alpha}}^{O}(\underbrace{\theta}_{w}) \rightarrow c_{\underbrace{\hat{O}}\underbrace{\check{O}}}{}^{O}(k,l,W) F_{\underbrace{\hat{\alpha}}}{}^{\hat{O}}(\underbrace{\hat{\theta}})\ F_{\underbrace{\check{\alpha}}}{}^{\check{O}}(\underbrace{\check{\theta}})$
- 1.Particle number n=0 mod N and k=0 mod N.
- $F_{\underbrace{\alpha}}^{J_a}(\underbrace{\theta}_{w})\ w \xrightarrow{\text{yields}} \infty \qquad -2\frac{\eta}{w} f_{abc} F_{\hat{\alpha}}{}^{J_b}(\underbrace{\hat{\theta}})\ F_{\check{\alpha}}{}^{J_c}(\underbrace{\check{\theta}})$

$F_{\underbrace{\alpha}}{}^{\varphi}(\underbrace{\theta}_{w})\ w \xrightarrow{\text{yields}} \infty \qquad F_{\underbrace{\hat{\alpha}}}{}^{\varphi}(\underbrace{\hat{\theta}}) F_{\underbrace{\check{\alpha}}}{}^{\varphi}(\underbrace{\check{\theta}})$

$F_{\underbrace{\alpha}}{}^{T}(\underbrace{\theta}_{w})\ w \xrightarrow{\text{yields}} \infty \qquad \frac{2\eta}{w^2} F_{\underbrace{\hat{\alpha}}}{}^{J_a}(\underbrace{\hat{\theta}}) F_{\underbrace{\check{\alpha}}}{}^{J_a}(\underbrace{\check{\theta}})$

Results.

2. Particle number n=0 mod N and k=1mod N, for the fields Current and energy momentum tensor.
3. Particle number n=1 mod N and k=0 mod N for the Fermi field.
4. Particle number n=1 mod N and k=1 mod N,

- again for the fermi field.

These results one can see in our paper in (BFK).

8.Deep Inelastic Scattering and Integrable models–Preliminary results.

- 1.We have prove that in 1+1 Integrable QFT's
- at small x the structure functions are factorized to functions of x and q^2.
- 2.We also prove that in the experimental results for the structure functions(HERA) has
- Factorized again to functions of x and q^2.
- 3.We have prove the Balog–Weisz conjecture that –at small x all structure functions in asymptotically free 1+1Integrable QFTS behavior as x^(–1)(ln)^(–2)!!

Deep Inelastic Scattering and Integrable models-Preliminary Results.

- 4.Bjorken Scaling for the structure function exist in some region of q^2 and this is feet with HERA experimental dates.
- 5.We have many interesting recommendation for the future experiments in order to clarify the behavior of the structure functions at small x and small q^2.

Conclusion.

- We investigate the rapidity clustering of exact multi-particle form-factors of SU(N) chiral Gross-Neveu model . For some examples of local fields , in particular the Noether current,
- The energy momentum tensor , the fundamental spinor field etc, we explicitly demonstrate the clustering or factorization phenomena. We connect this phenomena with Bjorken scaling and scattering.